Vectors and Scalars

Physical properties are quantified in one of two possible categories:

- **scalars**, which have magnitude only (but no direction),
- **vectors**, which have magnitude (size) and direction .

Typical scalar quantities include mass and temperature. Typical vector quantities include force and velocity. Since scalars are just numbers the rules for combining them are the same as those in ordinary arithmetic.

Example 1

Two masses of 3kg and 4kg are combined. What is the total mass.

Answer

Since mass is a scalar quantity, they are added: 3kg+4kg=7kg

The rules for combining vectors are more complicated. The methods concur with the mathematical definition of vectors (in matrix-vector arithmetic[1]).

In the following example we consider vector addition in one dimension. It is simple, but it serves to show that the addition of vector quantities is different to the addition of vector quantities; we need to also consider direction.

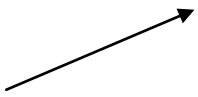
Example 2

Two moves of 3m west and then 4m east. State the total displacement.

Answer

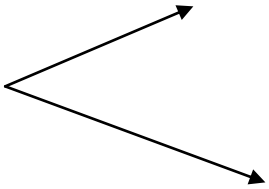
Because the 3m and 4m directions are opposite, we subtract rather than add. Measured in the east direction the displacement is 3m-4m =-1m east, or 1m west. The resultant displacement is 1m west.

Directions can be within a two-dimensional space. The vectors are represented by arrows; their magnitudes are represented by length of the arrows and their directions correspond to the directions of the arrows.

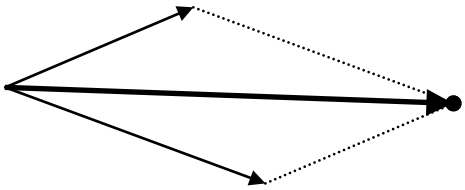


[1] www.mathematics.me.uk/matrices

In order to add two vectors, such as those illustrated in the following diagram.



The resultant can be found by completing the parallelogram, as illustrated in the following diagram.

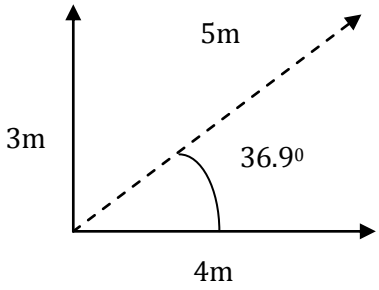


The resultant is the thick line. In general we can use the principles of trigonometry[2] to determine the magnitude and direction of the resultant.

Example 3

Two moves of 3m north and then 4m east. State the total displacement.

Answer



Because the 3m and 4m directions are at right angles we use Pythagoras's theorem to determine the length of the diagonal which gives 5m at an angle of 36.9^0 north of east.

[2] Trigonometry

Components of a vector

It is often more helpful to write a vector as the resultant of perpendicular components. Often there are reference axes – *x,y* in two dimensions and *x,y,z* in three dimensional space – to which we can place the vector components. In two-dimensional space we have the (1,0) = **i**, and (0,1)=**j**. In three-dimensional space we have three components (1,0,0)=**i**, (0,1,0)=**j** and (0,0,1)=**k**.

In two dimensions a vector can be written as the resultant of two perpendicular components.

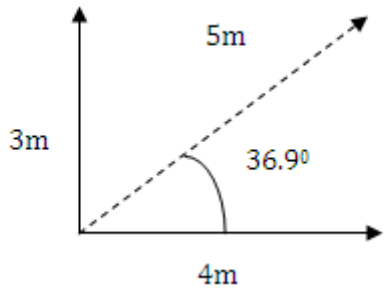
Example 4

Write the displacement of 5m at an angle of 36.9^0 north of east in component form

Answer

The east component can be found using trigonometry.

For example $\cos(36.9^0)$=*adjacent/hypotenuse*, this gives an *adjacent* of 4m. Similarly $\sin(36.9^0)$=*opposite/hypotenuse*, this gives an *opposite* of 3m.



Hence we can write the vector as (4m, 3m)=4**i**m + 3**j**m.